Curved space, monsters and black hole entropy

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Black hole entropy

A mystery of modern physics: $S_{\mathrm{BH}} = A/4$

Entropy given by area in Planck units. One bit per Planck area.

Bekenstein-Hawking: $T \sim R^{-1}$, $R \sim M$

$$S_{\mathrm{BH}} \sim \int \frac{dQ}{T} \sim \int R\, dM \sim R^2$$

The meaning of entropy

Entropy = log of number of microstates consistent with some macro condition.

$$S \sim \ln \{\#\text{ of microstates } N\}$$

Typically, entropy is *extensive*:

$$\ln\left(c^V\right) = V \ln c$$

The dimensionality of the Hilbert space describing a volume V is $\dim\mathcal{H} = c^V$ ($c = 2$ for qubit). $S \sim$ number of d.o.f.

- coarse graining
- loss of information

’tHooft bound

Exclude states whose energies are so large that they would have already caused gravitational collapse: $E < R$ (Hoop Conjecture)

Compute entropy; dominated by thermal configurations at large V:

$$S \sim T^3 R^3 \quad , \quad E \sim T^4 R^3$$

$E < R$ then implies

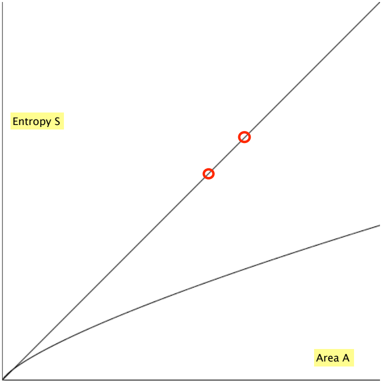
$$T \sim R^{-1/2} \quad \rightarrow \quad S < R^{3/2} \sim A^{3/4}$$

$$S < A^{3/4}$$

Note: Black hole *density* decreases with size. For any constant density $E(R) > R$ for sufficiently large R!

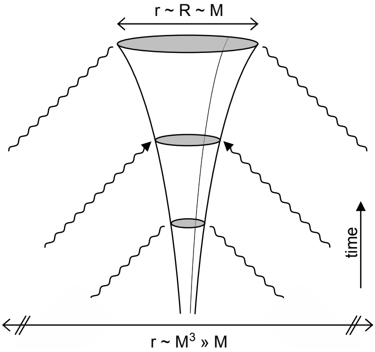
’tHooft bound

Ordinary matter satisfies $S < A^{3/4}$. What does this say about black holes, which have $S \sim A$? There is an entropy gap!



The number of ways to form a given black hole from a compact region of ordinary matter is much smaller than the number of possible black holes of that mass.

exp A distinct black holes?



There are exp A distinct states of Hawking radiation:

$$S = \int dQ/T \sim \int dM\, M \sim A$$

A black hole which slowly eats in-coming radiation can saturate $S \sim A$.

Negative binding energy

Consider N particles of mass m. In GR there is negative binding energy $\equiv \Delta$. The ADM mass can be less than Nm:

$$M = Nm - \Delta$$

In fact, one can achieve

$$\frac{M}{Nm} = \frac{Nm - \Delta}{Nm} \ll 1 \ .$$

This suggests that entropy to mass ratios can be very high. If the object becomes a black hole its area will scale as $A \sim M^2$.

Can the entropy equal or exceed A?

Curved space

Goal: Generalize 'tHooft analysis to curved space.

Construct configurations with large proper volume (fixed entropy density; large total entropy) but small ADM mass.

Configurations will be static (macroscopic moment of time symmetry; time reversal invariance) and satisfy Einstein constraints.

They comprise good initial data (matter + metric) for evolution using Einstein equations.

Curved space

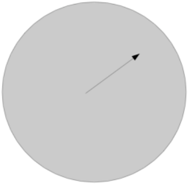
Consider spherically symmetric, but not necessarily static, distributions of matter

$$ds^2 = -g_{tt}(r,t)dt^2 + g_{rr}(r,t)dr^2 + r^2 d\Omega^2 \quad . \tag{1}$$

Energy within radius r:

$$M(r) = 4\pi \int_0^r dr'\, r'^{\,2} \rho(r') \quad , \tag{2}$$

where $\rho(r) = \rho(r, t_0)$ is the proper energy density (i.e., as seen by a stationary observer at r) on the initial time slice $t = t_0$.



ADM mass: $M \equiv M(R)$, where R is radius of object.

Curved space

Define

$$\epsilon(r) = 1 - \frac{2M(r)}{r} \quad , \tag{3}$$

Then, assuming the matter to be initially at rest w.r.t. our (r, θ, ϕ) coordinates, the metric on that slice is fully determined by

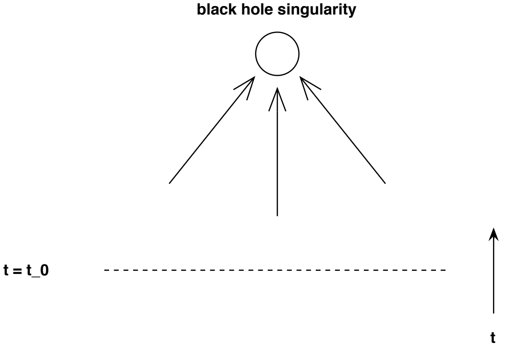
$$g_{rr}(r, t_0) = \epsilon(r)^{-1} \quad . \tag{4}$$

Each choice of $\rho(r)$ yields a good initial configuration.

Collapse

Each configuration satisfies the Einstein constraints and, as we will see, collapses to form a black hole.

Each leads to a *distinct* black hole internal state; our goal is to count them.



Curved space

Entropy: assume covariantly conserved entropy current $j^\mu : \quad j^\mu{}_{;\mu} = 0$. Stokes theorem:

$$S_\Sigma \;=\; \int_\Sigma d^3x\,\sqrt{\gamma}\,s \;=\; \text{constant} \;\;, \tag{5}$$

where integral is over a constant time slice Σ with induced metric γ and unit normal $n^\mu \sim (\partial_t)^\mu$.

$s = -j^\mu n_\mu$ is the proper entropy density (as seen by a stationary inertial observer). In our coordinates, $s(r) = j^0(r, t_0) g_{tt}(r, t_0)^{1/2}$.

Total entropy on the initial time slice t_0 is

$$S = 4\pi \int_0^R dr\, r^2 \epsilon(r)^{-1/2} s(r) \;\; . \tag{6}$$

Curved space: key formulae

Total ADM mass (includes negative binding energy):

$$M \equiv M(R) = 4\pi \int_0^R dr' \, r'^2 \rho(r') \ ,$$

Total entropy: ($\epsilon(r) = 1 - 2M(r)/r$)

$$S = 4\pi \int_0^R dr \, r^2 \epsilon(r)^{-1/2} s(r) \ .$$

Both $\rho(r)$ and $s(r)$ are proper densities (as seen by stationary inertial observer).

E.g., thermal photons: $\rho(r) \sim T(r)^4, s(r) \sim T(r)^3$ or $s(r) \sim \rho(r)^{3/4}$.

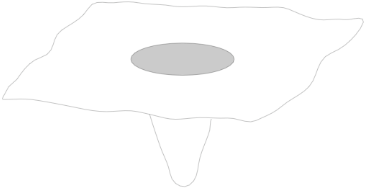
Maximize S while holding M fixed.

Curved space: monster

The *proper* volume of our object is

$$V_p = 4\pi \int_0^R dr\, r^2 \epsilon(r)^{-1/2} \quad . \tag{7}$$

Fixed ADM mass (surface area); potentially infinite proper volume and total entropy, keeping entropy and energy *densities* finite.

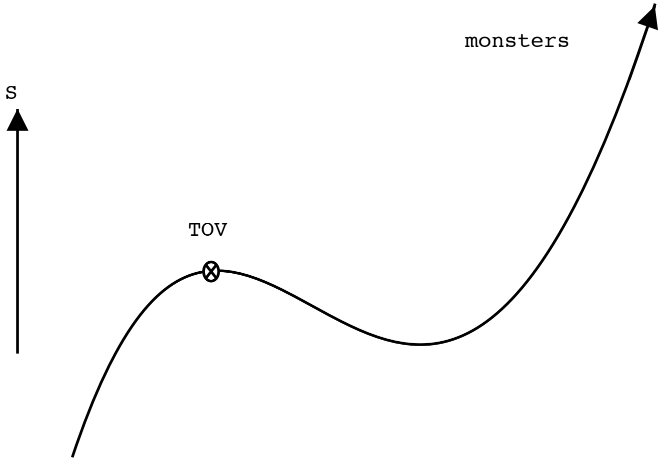


Trick: adjust $\epsilon(r) = 1 - 2M(r)/r \approx 0$ in some large region.

See also Sorkin, Wald and Zhang (1981)

Entropy extrema

Hold ADM mass M fixed, extremize entropy S.



Example monster: blob

Suppose: small core of radius r_0, mass M_0 and density profile

$$\rho(r) = \rho_0 \left(\frac{r_0}{r}\right)^2 \quad (r_0 < r < R) \ . \tag{8}$$

Then

$$M(r) = M_0 + 4\pi\rho_0 r_0^2 (r - r_0) \ . \tag{9}$$

Let $8\pi\rho_0 r_0^2 = 1$ so that

$$\epsilon(r) = \epsilon_0 \left(\frac{r_0}{r}\right) \ , \tag{10}$$

where $\epsilon_0 = 1 - 2M_0/r_0$.

Example monster

Total entropy (neglecting the small core region $r < r_0$):

$$S \sim 4\pi \int_{r_0}^{R} dr\, r^2 \left(\frac{r}{r_0\epsilon_0}\right)^{1/2} \rho^{3/4} \sim \frac{\rho_0^{3/4} r_0}{\sqrt{\epsilon_0}} R^2 \ . \tag{11}$$

1) Area scaling has been achieved.

2) S can be made as large as desired by taking ϵ_0 small.

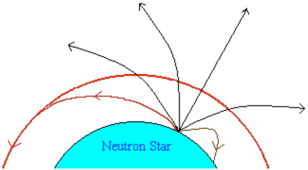
3) Can obtain faster than A scaling by taking $\epsilon(r)$ to approach zero faster than $1/r$.

Escape angle and the fate of monsters

Trapped surface? No.

Horizon? Eventually, yes.

Future of monster interior does not include future null infinity $\mathcal{I}^+$

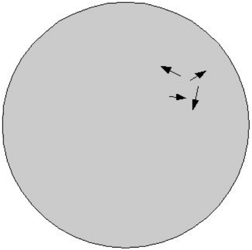


Critical escape angle $\equiv \theta_c$.
$\theta_c^2 \sim \epsilon(r) = 1 - 2M(r)/r$.

Escape angle and the fate of monsters

All monsters must have $\epsilon \approx 0$ (and $\theta_c \approx 0$) in large subregion.

Hence, will inevitably evolve into a black hole: mean-field analysis shows net inward energy flow.



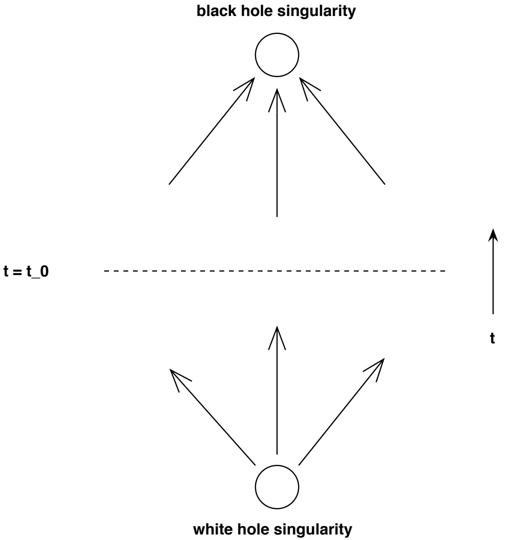
Will also evolve into a black hole if *time-reversed*.

So, cannot form from good initial data (without intervention): requires initial *white hole* singularity.

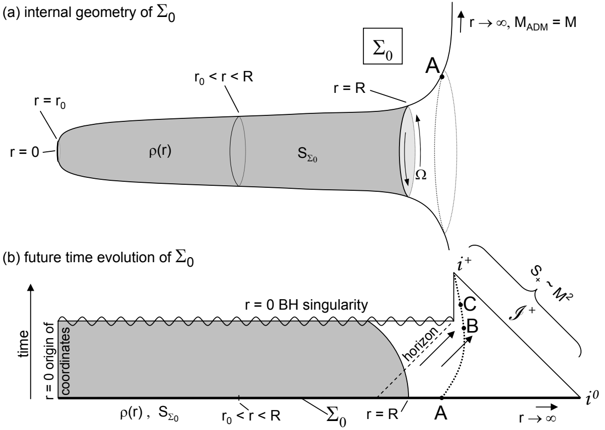
Isolated monster

Matter ejected from white hole reaches turning point (object is gravitationally bound) and recollapses into black hole.

Note time-reversal invariance.



Monster spacetime



Build a monster

Assume arbitrarily advanced civilization, constraints only from fundamental physics. Can one construct a monster?

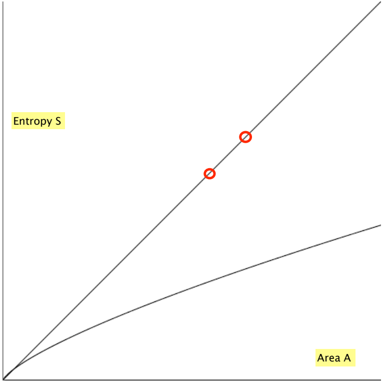
Buildability condition: require configuration to be no closer than a thermal wavelength $\lambda \sim \rho^{-1/4}$ from its Schwarzschild radius.

$$r\epsilon(r) \; > \; r - 2M(r) \; > \; \lambda$$

implies the bound $S < A$.

Can't build monsters with entropy $S > A$.

Entropy gap?



Perhaps this closes the entropy gap?

Most entropic pre-black hole state is either a slightly smaller black hole, or a monster state with entropy A.

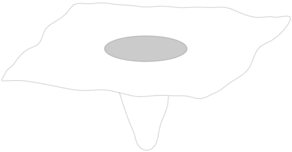
Build a monster: tunneling

Initial data: collapsing spherical shell of energy, ADM mass M, other quantum numbers $Q = J = 0$.

Same quantum numbers as monster.

There must be a nonzero probability for our initial data to evolve (tunnel) into the monster state, even those with $S >> A$!

Otherwise, $\exists$ new selection rule forbidding certain transitions between states with same quantum numbers.



How big is the Hilbert space for gravity?

Let Φ = matter fields; g = geometry or metric.

$$(\Phi, g) \ \in \ \mathcal{H}$$

Good semiclassical evidence for

$$\dim\mathcal{H} \ >> \ \exp A$$

Black hole entropy and microstates

It is claimed that black hole entropy counts the number of possible microstates of the hole (Strominger and Vafa):

$$\#\text{ of microstates} \sim \exp A/4$$

This is problematic if tunneling to monster states is allowed: a monster-like configuration might lurk behind the horizon, with the possibility of many more microstates.

- Hyper-entropic states?
- Remnants?

AdS/CFT

It is plausible that monster configurations exist in AdS.

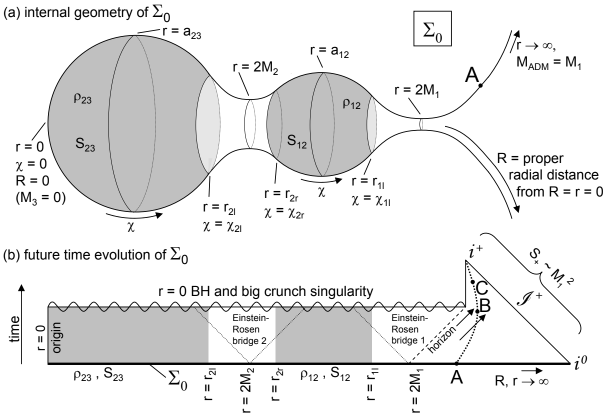
Can they be described by the boundary CFT? Not enough degrees of freedom!

Related discussion: inflationary pocket universes in AdS/CFT

Freivogel, Hubeny, Maloney, Myers, Rangamani and Shenker, JHEP 0603:007,2006.

Conclusion: new selection rule in quantum gravity!

Einstein-Rosen gluing



Conclusions

Using curved space, one can (mathematically) construct objects with more entropy than a black hole of equal mass.

Implications for black hole entropy, holography, AdS/CFT.

Whether one can, in principle, physically construct such objects is unclear.